NUMERICAL INVESTIGATION OF SPALLING RUPTURE IN COPPER

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The cumulative damage concept, which takes the change in structure of the material during deformation as basis, has recently been utilized extensively in the investigation of rupture in both static and dynamic problems. It should here be kept in mind that it is impossible to describe all processes in solids by simply the dynamics of the stress state, and additional account must be taken of processes occurring in the solid, the additional kinetics that takes account of the reverse influence of the rupture on the stress and strain fields [1]. Under rupture conditions with dynamic loads, the stress state ordinarily varies substantially at distances considerably exceeding the size of the brokenness, being formed in the initial stage of the rupture process, and such a medium can, on the average, be considered as continuous, i.e., the development of individual cracks need not be followed, nor their total effect taken into account. It is included in the origination of a large number of cracks or pores and in the diminution of the strength as a result.

On the basis of investigations [3, 4], a rupture model is developed in [2] that takes account of the formation of pores in the material and of their further growth under the effect of tensile stresses. This model is used below for a numerical analysis of the spalling rupture in copper and the results of the calculations are compared with experimental data [5, 6].

1. The computational model developed in [2] is constructed from two component parts: 1) equations describing the laws of brokenness generation and their development (a specific microdefect structure is taken, viz., pores of spherical shape); 2) equations governing the elastic–plastic behavior of the material with stress relaxation and the change in mechanical properties (strength reduction) because of the appearance and growth of a large quantity of cavities (pores) taken into account.

The equation for the change in relative pore volume with time can be presented in the form [2-4]

$$V_n = V_{n0} \exp \frac{3(p_s - p_{g0})\Delta t}{4\eta} + 8\pi \dot{N} R_n^3 \Delta t, \quad (1.1)$$

where V_{n0} is the relative pore volume at the time t; p_s, pressure in the solid; p_{g0}, η, certain constants; $\dot{N}$, rate of micropore generation per unit volume; and R_n, distribution parameter of the newly formed pores during the time from t to t + Δt.

To analyze the material behavior during deformation and rupture, we use the theory of plasticity of porous bodies.

The body being investigated (or the system of bodies) is separated into parts (a certain standard volume) whose size is sufficiently large as compared with the characteristic size of the cracks being generated. On the other hand, the size of these parts should be sufficiently small compared to the characteristic scales of the change in macroscopic stresses in the body under investigation. The stress field in the standard volume is considered independent of the coordinates. A cell of the finite-difference mesh is taken as standard volume when using numerical methods, and the kinetics of the stress–strain state and the kinetics of development of brokenness are followed therein.

2. Experiments on the collision of flat slabs of copper were performed in [5] in order to determine the three stress quantities governing different degrees of rupture. Impacts on a fixed plate (target) were executed at different velocities for approximately the same plate-impactor thickness. The specimen microstructure was then analyzed carefully, and the degree of rupture of the continuity was determined then.

The described experiments were modelled on an electronic computer by solving the

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TABLE 1

Variant No.	u, m/sec	h_1, cm	h_2, cm	σ, GPa	V_n, %	
1	53,34	0,312	0,493	0,95	$3\cdot10^{-2}$	B
2	99,36	0,292	0,597	1,77	4	C
3	122,22	0,289	0,599	2,18	11,6	C
4	128,02	0,350	0,650	2,28	14,7	C–D
5	156,36	0,307	0,617	2,78	29,2	D
6	176,78	0,335	0,604	3,15	39,5	D
7	205,44	0,302	0,587	3,66	45	D
8	221,89	0,305	0,587	3,95	>65	D
9	253,59	0,297	0,610	4,51	>89	D
10	38,1	0,150	0,637	0,68	None	A
11	153	0,155	0,551	2,72	22,9	C–D

one-dimensional elastic–plastic problem of plate collision. Errors introduced into the results by the finite-difference scheme of the solution itself were reduced in the numerical investigation of models of the simplest (one- and quasi-one-dimensional) experiments, for whose realization there is no necessity to solve complex boundary value problems. Performing such computations, and comparing their results with the results of physical experiments permit the purest estimation of the model selected. Numerical results are obtained for the following data for copper [3, 5]: density ρ_0 = 8.86 g/cm^3, shear modulus G = 41 GPa, volume compression modulus K = 114.9 GPa, yield point σ_T = 0.3 GPa, constants in the relationship (1.1) – p_{n0} = 0.5 GPa, p_{g0} = 0.5 GPa, η = 20 Pa·sec. Analysis of the influence of the finite-difference scheme parameters (the spacing in the space and in the time coordinates) on the results of the numerical experiment preceded the computations. The quantities mentioned were selected in such a way that their influence on the results of solving the problem was minimal. In particular, it was clarified that it is expedient to compare the relative cavity volume only for the identical spacing in the space coordinate. The results of computations and their comparison with experimental data are represented in Table 1 and in Figs. 1-4. In Table 1 the u is the velocity of the plate-impactor, h_1 is its thickness, h_2 is the specimen thickness (target), σ is the amplitude of the compressive stress pulse, V_n is the relative pore volume in the spall plane. The plane in the target on which the maximum value of the damage takes place is taken as the spall plane. The degree of damage defined in [5] is in the last column of Table 1. For qualitative characteristics of the degree of damage in Table 1 we take the following notation: A – no spoilage of the continuity; B – formation of cracks or pores exposable on a microsection, in the specimen (initial spall); C – formation of a large number of cavities in the target, part of which grow to a magnitude exceeding the grain diameter (intermediate spall); D – beginning of the formation of a rupture surface (total spall).

Time is plotted along the abscissa axis in Figs. 1-3, and the velocity of the specimen free surface along the ordinate axis; the change with time of the volume fraction of cavities

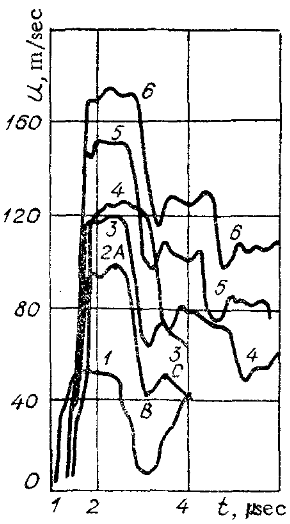

Fig. 1.

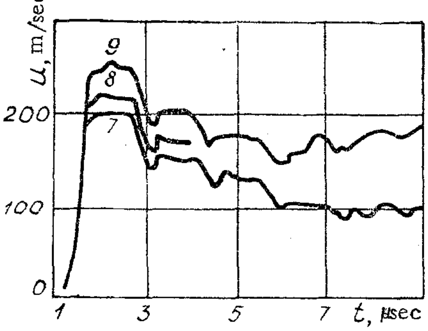

Fig. 2

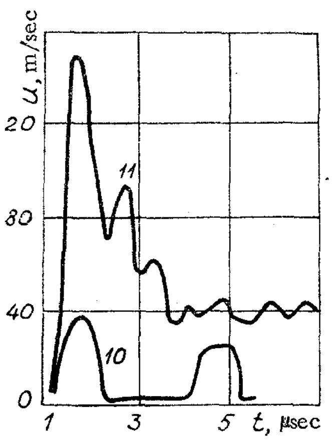

Fig. 3.

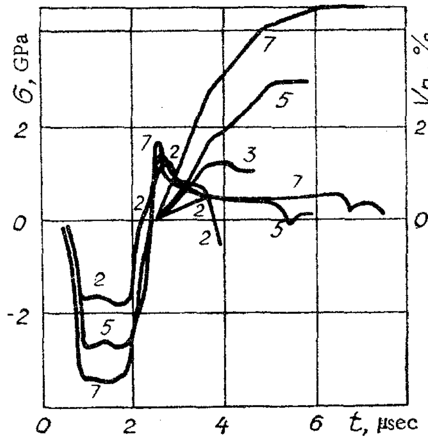

Fig. 4.

and the stress in the spall plane is represented in Fig. 4 for different collision conditions. The number of the curves in Figs. 1-4 corresponds to the numbers of the versions in Table 1.

The compressive stress wave originating in the target (its magnitude is determined by the collision velocity) is reflected from the free target surface and a tensile stress domain (Fig. 4) is formed in the target during interaction with the opposite unloading wave going from the free surface of the impacting plate. When the tensile stresses (pressure) reaches the threshold value p_{n0} pores start to form in the target. As the pore concentration grows, unloading of the material occurs in the zone of the growing cavities, and the magnitude of the tensile stresses is lowered (Fig. 4), which causes propagation of a disturbance (a spall pulse) on both sides of the rupture zone. When the spall pulse reaches the free surface of the target, its velocity grows abruptly (see Figs. 1-3), and in the absence of damage the velocity growth occurs with the arrival of the stress pulse already reflected from the contact surface of the impactor and the target (curve 10 in Fig. 3).

The following can be noted by analyzing the nature of the stress change in the plane of maximum damage (Fig. 4) and comparing it with the cumulative damage history. The rate of accumulation of the relative volume of cavities rises substantially as the collision velocity increases. If the relative volume of the cavities is less than 12-15%, then compressive stresses develop after the tensile stresses in the spall plane and a certain pore compression occurs (curve 3 in Fig. 4). Large damage levels (to 30%) accumulate in two stages: it is seen in curve 5 in Fig. 4 that the relative volume of cavities in the first stage (to 4 μsec) grows at a velocity very much greater than in the second stage. The nature of the stress change in the spall plane becomes qualitatively different: the duration of the action of the tensile stresses increases (curve 5 in Fig. 4), and pore compaction does not occur. This is associated with the fact that as the stress pulse intensity increases, the rate of pore formation grows, whereupon a more powerful spall pulse is formed which on being reflected from the free surface of the target arrives at the rupture zone in the form of tensile stresses and contributes to the further growth of the damage. Only a negligible increase in the tensile stress duration in the plane of maximum damage [2] occurs for slight damage. For damage levels greater than 30% (curve 7 in Fig. 4), three phases can be noted on the graph of the growth in the relative cavity volume, wherein the growth rates differ substantially, where the growth rate of the relative cavity volume diminishes from stage to stage. At still higher damage levels, their growth rate is retarded somewhat after the first phase, and then grows constantly, hence, the duration of the tensile stress phase in the spall plane starts to diminish. The features noted above evidently correspond to different stages in the rupture process.

TABLE 2

Variant No.	2	3	4	5	6	7	8	9	11
σ_p, GPa	1,12	1,12	1,08	1,10	1,14	1,16	1,16	1,16	1,53
R	0,52	0,608	0,63	0,72	0,74	0,8	0,795	0,8	0,617

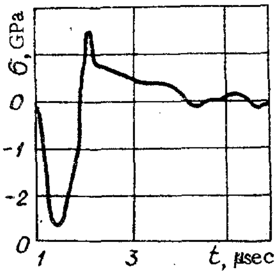

Fig. 5.

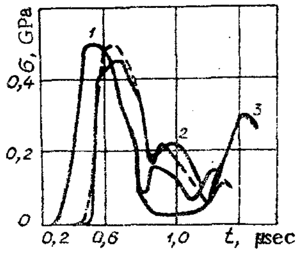

Fig. 6.

The ratio between target thickness and plate-impactor thickness is approximately two in versions 1-9. Versions 10 and 11 correspond to impact on the same target, as in the first cases, however, the projectile thickness is less. The results of these computations are represented in Figs. 3 and 5. Here the duration of the compressive stresses is on the order of 0.8 μsec as against 1.5-1.9 μsec in the first cases. It is seen from the results that the collision velocity (stress) required to obtain a definite damage level, should be greater in this case, as is known for earlier investigations of the spall process [1].

As is noted above, the change in velocity of the specimen free surface as compared with its motion in the absence of rupture is due to the propagation of a disturbance being formed in the zone of the growing pores. In this connection, the critical magnitude of the tensile stresses [7, 8], as well as the size of the rupture level [9] are often determined from direct processing of the free surface velocity profile.

The magnitude of the rupturing stress is determined according to the following relationship

$$\sigma_p = 0.5\rho_0 c_0 (u_A - u_B), \quad (2.1)$$

where $u_A - u_B$ is the difference between the maximal and minimal velocities of free surface motion (Fig. 1), ρ_0 is the initial density, and c_0 is the speed of sound.

The results of calculating σ_p by means of (2.1) are represented in Table 2. It is seen that for versions 2-9 the stress σ_p depends weakly on the impact velocity and its value is ≈1.1 GPa. This value is between those determined in [5] by the limit of the initial spall, equal to 0.6 GPa, and the limit of intermediate spall, equal to 1.6 GPa. For thinner plate-impactors (version 11) σ_p = 1.53 GPa. For a still greater increase in the plate-impactor thickness, the value of σ_p diminishes. Thus, for the impact by a plate h_1 = 2 cm on a target h_2 = 4 cm we have σ_p = 0.83 GPa, whole for the impact of a plate h_1 = 5 cm on a target h_2 = 10 cm, we have σ_p = 0.76 GPa. In the last two cases, a decrease in the quantities σ_p is observed as the impact velocity increases (the greatest values are presented). This indicates that (2.1) must be used with extreme care for thick plate-impactors.

The quantity

$$R = u_C/u_A \quad (2.2)$$

is proposed in [9] for utilization to estimate the degree of rupture.

The results of calculating R by means of (2.2) are represented in Table 2. Comparing the quantity R with the relative cavity volume V_n (Table 1) shows that the value of R grows as V_n increases although the damage level is below that determined. If the damage grows in two stages and more, then R will correlate less and less with the magnitude of the relative cavity volume. This is explained by the fact that in these cases pore formation and growth continues even after the time corresponding to the point C.

To study the properties of materials under dynamic loads, a somewhat different scheme of the experiment is used than that investigated above. A plate of the material being studied, which abuts on one side a material of lower acoustic hardness, is loaded by the impact of a plate, as in [5], on the other surface. By using a dielectric sensor, say, the stress on the interface between the material under investigation and the material of lower acoustic hardness is recorded. The solution of such a problem and its comparison with numerical and experimental results [6] are represented in Fig. 6. In this case a plate-impactor from 0.622-mm-thick copper exerts an impact on a fixed specimen of 1.587-mm-thick copper. The specimen abuts a material of organic glass type on the other side. The physicomechanical characteristics for copper were taken exactly as in the previous computations, and for organic glass as: $\rho_0 = 1.2$ g/cm³, K = 8.95 GPa, G = 0.96 GPa. The present solution with the rupture process taken into account is denoted by 1 in Fig. 6, without taking rupture into account by 3, experimental data [6] by 2, and numerical data [6] by dashes. The rise in stress around 0.8 μsec is explained by the arrival of a disturbance at the interface between the copper and the organic glass which is due to the origination and growth of pores in the target. Results of solving this problem agree qualitatively and in order of magnitude with the results in [6], which can also be considered as confirmation of the rupture model being used. Existing discrepancies, particularly the time shift, can be explained by differences in either the target thickness or the properties of the material.

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LIMITING STRAINS OF THE DYNAMIC FRACTURE OF CYLINDRICAL SHELLS

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The implementation of the fracture criteria of [1, 2] with application to rigidly plastic cylindrical shells expanding under the action of detonation products is discussed. The experimental results for the determination of the limiting strains of the shells are discussed. The problem of the limiting strains of cylindrical shells under the action of detonation products expanding in equilibrium has been discussed in [2-4]. Numerical and analytic solutions describing the process of deformation of a cylindrical rigidly plastic shell have been obtained in [4, 5].

We shall consider some criteria and conditions for the fracture of rigidly plastic shells. An outer zone with a mixed stress state ($\sigma_r < 0$, $\sigma_\theta > 0$) and an inner zone in which a state of hydrostatic equilibrium ($\sigma_r < 0$, $\sigma_\theta < 0$) is realized arise in a cylindrical shell acted on by an intense internal load [4]. Here σ_r and σ_θ are the radial and tangential components of the stress tensor. The condition satisfied on the boundary of both zones has the form $\sigma_\theta = 0$. Its use together with the plasticity condition ($\sigma_\theta - \sigma_r = \varkappa Y$) and the expression for the radial stress [5] in the initial position ($a = a_0$, $b = b_0$) will give the coordinate of the boundary $\sigma_\theta = 0$:

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